# Injective Functions

$$isInjective(f) \iff \forall s_1, s_2, t \bullet (s_1 \in S \land s_2 \in S \land t \in T) \Rightarrow ((s_1, t) \in f \land (s_2, t) \in f \Rightarrow s_1 = s_2)$$

If $f$ is a ***partial injection***, we write: $f \in S \nrightarrow T$

- e.g., $\{ \varnothing, \{(1, \mathbf{a})\}, \{(2, \mathbf{a}), (3, \mathbf{b})\} \} \subseteq \{1, 2, 3\} \nrightarrow \{a, b\}$
- e.g., $\{(1, \mathbf{b}), (2, a), (3, \mathbf{b})\} \notin \{1, 2, 3\} \nrightarrow \{a, b\}$
- e.g., $\{(1, \mathbf{b}), (3, \mathbf{b})\} \notin \{1, 2, 3\} \nrightarrow \{a, b\}$

If $f$ is a ***total injection***, we write: $f \in S \rightarrowtail T$

- e.g., $\{1, 2, 3\} \rightarrowtail \{a, b\} = \varnothing$
- e.g., $\{(2, d), (1, a), (3, c)\} \in \{1, 2, 3\} \rightarrowtail \{a, b, c, d\}$
- e.g., $\{(\mathbf{2}, d), (\mathbf{1}, c)\} \notin \{1, 2, 3\} \rightarrowtail \{a, b, c, d\}$
- e.g., $\{(2, \mathbf{d}), (1, c), (3, \mathbf{d})\} \notin \{1, 2, 3\} \rightarrowtail \{a, b, c, d\}$

# Surjective Functions

$$isSurjective(f) \iff \underline{\text{ran}(f)} = \underline{T}$$

If $f$ is a ***partial surjection***, we write: $f \in S \twoheadrightarrow T$

- e.g., $\{ \{(1,\mathbf{b}),(2,\mathbf{a})\},\{(1,\mathbf{b}),(2,\mathbf{a}),(3,\mathbf{b})\} \} \subseteq \{1,2,3\} \twoheadrightarrow \{a,b\}$
- e.g., $\{(2,\mathbf{a}),(1,\mathbf{a}),(3,\mathbf{a})\} \notin \{1,2,3\} \twoheadrightarrow \{a,b\}$
- e.g., $\{(2,\mathbf{b}),(1,\mathbf{b})\} \notin \{1,2,3\} \twoheadrightarrow \{a,b\}$

If $f$ is a ***total surjection***, we write: $f \in S \twoheadrightarrow T$

- e.g., $\{ \{(2,a),(1,b),(3,a)\},\{(2,b),(1,a),(3,b)\} \} \subseteq \{1,2,3\} \twoheadrightarrow \{a,b\}$
- e.g., $\{(\mathbf{2},a),(\mathbf{3},b)\} \notin \{1,2,3\} \twoheadrightarrow \{a,b\}$
- e.g., $\{(2,\mathbf{a}),(3,\mathbf{a}),(1,\mathbf{a})\} \notin \{1,2,3\} \twoheadrightarrow \{a,b\}$

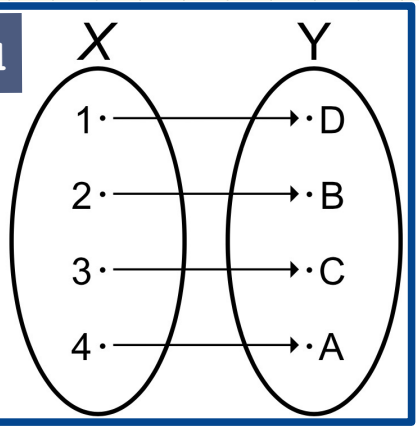
# Bijective Functions

*f* is ***bijective***/***a bijection***/*one-to-one correspondence* if *f* is ***total***, ***injective***, and ***surjective***.

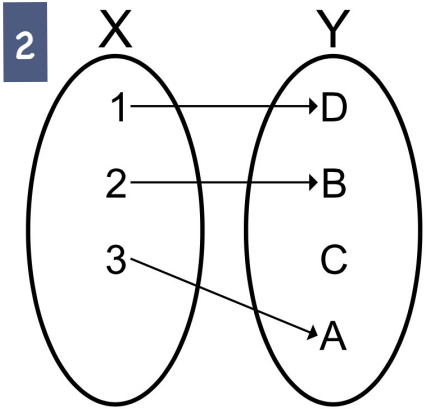
- e.g., $\{1,2,3\} \rightarrowtail\!\!\!\!\!\twoheadrightarrow \{a,b\} = \varnothing$
- e.g., $\{\ \{(1,a),(2,b),(3,c)\},\{(2,a),(3,b),(1,c)\}\ \} \subseteq \{1,2,3\} \rightarrowtail\!\!\!\!\!\twoheadrightarrow \{a,b,c\}$
- e.g., $\{(\mathbf{2},b),(\mathbf{3},c),(\mathbf{4},a)\} \notin \{1,2,3,4\} \rightarrowtail\!\!\!\!\!\twoheadrightarrow \{a,b,c\}$
- e.g., $\{(1,\mathbf{a}),(2,b),(3,c),(4,\mathbf{a})\} \notin \{1,2,3,4\} \rightarrowtail\!\!\!\!\!\twoheadrightarrow \{a,b,c\}$
- e.g., $\{(1,\mathbf{a}),(2,\mathbf{c})\} \notin \{1,2\} \rightarrowtail\!\!\!\!\!\twoheadrightarrow \{a,b,c\}$

# Exercise

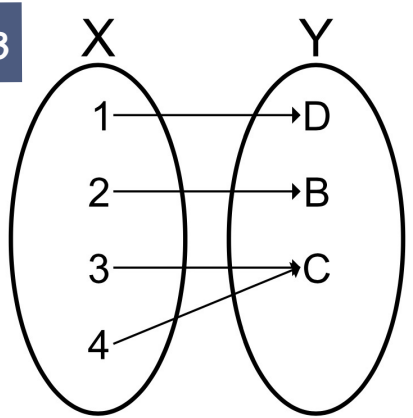
1



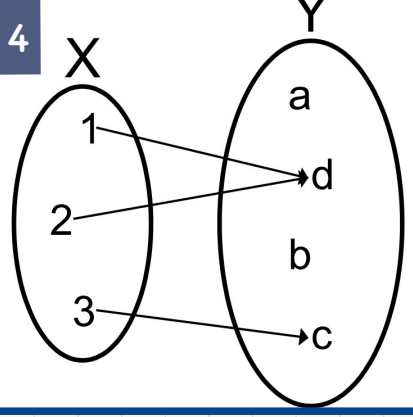

2




3




4




| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| partial | | | | |
| total | | | | |
| injection | | | | |
| surjection | | | | |
| bijection | | | | |